A Streamlined Real Options Model for Real Estate Development

by

Baabak Barman   And   **Kathryn E. Nash**
B.A., Applied Mathematics, 2003   B.A., Economics, 1998
University of California, Berkeley   Wesleyan University

Submitted to the Department of Urban Studies and Planning
in Partial Fulfillment of the Requirements for the Degree of

Master of Science in Real Estate Development

at the
Massachusetts Institute of Technology
September 2007

Signature of Author_____
Baabak Barman
Department of Urban Studies and Planning
July 27, 2007

Signature of Author_____
Kathryn E. Nash
Department of Urban Studies and Planning
July 27, 2007

Certified by _____
David Geltner
Professor of Real Estate Finance, Department of Urban
Studies and Planning
Thesis Supervisor

Accepted by _____
David Geltner
Chairman, Interdepartmental Degree Program in
Real Estate Development

Abstract

This thesis introduces a streamlined model that incorporates the value of the real options that exist in real estate development projects.[1] Real options add value to a project by providing developers with flexibility to minimize downside risk or take advantage of upside potential as conditions change from deterministic expectations. Though developers currently incorporate this value into their decision making using intuition and judgment, the model presented here provides a tool with which developers can value options in a rigorous and quantitative fashion. Though the model should not be used as a comprehensive land residual model, it serves as a powerful proof of concept for real options analysis in the field of real estate. Further, it can be used to measure the relative value and risk of projects with and without real options.

The model is based on both the traditional economic and the more recent engineering real options methodologies. Both approaches have been applied to real estate development projects, but have not yet caught on due to their newness and complexity. The streamlined model incorporates the elements of both methodologies that are most applicable to current development practice. In addition, the model is simplified and tailored to existing valuation techniques. The added benefit of this "hybrid" approach is that it reduces the learning curve associated with real options analysis so as to encourage its adoption in the real estate field in the short term.

The model uses Monte Carlo simulations in Excel and is targeted towards specific options scenarios commonly faced by developers; specifically, the options to phase a project, choose among multiple uses, and defer development. A case study demonstrates the model, and compares the results of building two phased buildings versus a single larger building on the same site. The results show that the phased program results in less risk and a higher expected net present value than the single building program, while the option to defer development adds significant value to both programs.

---

[1] This model is available upon request to the authors.

Acknowledgements

We would like to sincerely thank Professor David Geltner for the expert advice and guidance that he provided during this thesis process. We would also like to thank him for introducing the engineering approach to real options analysis in the Advanced Topics in Finance course. It was the pragmatic and multidisciplinary spirit of the engineering approach that initially sparked our interest.

We would like to thank Professor Richard de Neufville and Michel-Alexandre Cardin for furthering our understanding of the engineering approach and for guiding us as we incorporated the methodology into our thesis.

We are grateful to the real estate practitioners who took the time to be interviewed for this thesis. These interviews broadened our perspective and opened our eyes to how real estate professionals evaluate and make decisions about development projects in the "real world".

We would like to thank our fellow students, as well as the staff, at the CRE for making the past year truly memorable and for enriching our educational experience. We are especially grateful to Shuichi Masunaga and Matthew Lister for joining us on our journey down the path of real options analysis; it was an adventure.

Finally, we would like to sincerely thank our family and friends for their love, support, and understanding during our year at the CRE. We could not have done it without you!

Table of Contents

List of Figures

List of Tables

1. Introduction

1.1. Background and Purpose

This thesis presents a financial economic model that accounts for the uncertainty in real estate development projects and values the real options a developer has at his disposal in a rigorous and quantitative way. The model is designed to be as simple as possible, and to dovetail with how developers currently evaluate projects. By customizing and streamlining the model, we hope to overcome the barriers that currently exist in applying real options analysis (ROA) to real estate development, and provide developers with a tool to help them better understand the rewards associated with real options.

Real options analysis is an important topic in real estate due to the nature of the development environment. In particular, one of the foremost characteristics of the development process is the uncertainty that exists over the course of a project. When a developer initially conceives of a project, he or she has a base set of assumptions upon which an expected financial return for that project is calculated. However, as reality is sure to deviate from these assumptions, the actual financial return that the project will generate is unknown.

It is the uncertainty around a project's financial return that causes risk. Per Geltner and Miller (2007), risk is the possibility that future investment performance may vary over time in a manner that is not entirely predictable at the time when the investment is made. Because risk impacts financial returns, managing it plays a key role in the development process. As one developer expressed; "the control of risk is the essence of real estate development."[2]

Flexibility allows a developer to control risk. Each source of flexibility is, in technical terms, a "real option". An option is defined as the right, but not the obligation, to take some course of action in the future; they exist whenever two conditions are met: (i) new information will arrive in the future; and (ii) when it arrives, this news will affect decisions.

---

[2] Neel Teague, 2007.

Using the model presented in this thesis, we will value the options that allow a developer to respond to risk.

1.2. Methodology

The methods used in this thesis were qualitative and quantitative. The qualitative portion included a comprehensive series of industry interviews, as well as a review of existing work in the field of real options and its applications to real estate. The quantitative portion of the thesis consisted of developing a "hybrid" real options model. We demonstrate how the model works by using a real-world case study.

The interview portion of the thesis followed a semi-structured format. It consisted of discussions with eighteen real estate professionals in the fields of finance and development who worked for private development groups, REITs, investment management firms, and larger integrated real estate services firms. Many of the interviewees had development experience at the regional, national, and even international levels, often across multiple product types. Our questions were focused on understanding how the development community currently perceives, mitigates, and values risk and flexibility in development projects. An analysis of the interviews is presented in Chapter 2.

Concurrent with our interviews, we conducted a thorough literature review. The results of the literature review are presented in Chapters 3 and 4. Chapter 3 starts with a basic review of real options concepts. It goes on to describe the types and characteristics of real options that exist in real estate. Chapter 4 discusses the two real options methodologies that have been used to evaluate development projects: options valuation theory (OVT) and the engineering approach. The chapter includes a discussion of the strengths and weaknesses of these methodologies for evaluating development projects.

In Chapter 5 we present our model, which is based on a "hybrid" of both the option valuation theory (OVT) and engineering methodologies. The model is a simple and

transparent spreadsheet that can be readily applied to common situations faced by the average developer.

Finally, we apply theory to practice by way of two real world case studies, only one of which is reported in this thesis due to space and time constraints.[3] The reported case study focuses on a simple phasing option in which the developer has the choice to develop one single building, or two smaller buildings in phases.[4] This type of option, besides being common in the development world, serves as a "proof of concept" to demonstrate the nature and functionality of the model. The case study, which is presented in Chapter 6, demonstrates the use of the model and interpretation of its results. Following the case study, we will present our conclusion in Chapter 7.

---

[3] The second case study presents a simple example of a rainbow American call option in which the developer is considering changing the intended use of a project from office to retail.
[4] Both programs yield the same net rentable square footage.

2. Analysis of Industry Interviews

We conducted eighteen semi-structured interviews with real estate professionals during the months of June and July. The purpose of the interviews was to understand how practitioners evaluate development risk and value development projects. The interviews also served as an assessment of current industry methods of accounting for uncertainty and valuing flexibility. This chapter presents the findings of the semi-structured interviews.

2.1. Real Estate Development Requires Risk Mitigation

Based on discussions with interviewees, the sources and types of risk in the development process seemed limitless. For instance, projects can be adversely affected by issues ranging from cranes not fitting on a site, to environmental contamination discovered during excavation, to terrorist attacks that have ripple effects on the entire real estate industry. To compound the problem, each project has a dramatically different risk profile due to the heterogeneous nature of real estate. Taking into account the myriad of possible problems and uncertainties, it becomes clear that developing a model that takes into account all risks in real estate is, as one interviewee put it, "an exercise in futility".

It is in this capacity that a developer's ability to categorize, quantify, and mitigate risk adds significant value. Using her experience, skills, and judgment, a sophisticated developer can narrow down a daunting array of possible uncertainties into a crucial set of key factors that ultimately determine the project's value. Some risk factors can be fully quantified and mitigated by way of the developer's past experience and skill set. As an example, one interviewee stated that she was able to secure not only entitlements, but floor area ratio ("FAR") bonuses that allowed her to build more than indicated in the original zoning, in ninety percent of her development projects. Under these circumstances, entitlement risk, which is normally considered the riskiest part of the development cycle, can be considered a minor risk.

While many project level risks can often be mitigated by the developer's experience and skills, many market level risks, such cap rates, are much more difficult to mitigate. In such cases, the developer's approach is to look at a range of possible outcomes, understand their implications, and compare the assessed risk to the projected return in order to decide whether the project is worth pursuing. Once the developer has gone through the exercise of narrowing down key risks, the effects of those risks can be quantified and expressed as a single output: the value of the built property. In this fashion, a model can approximate the sum of crucial risk factors in a development project by condensing uncertainty into one value: the value of the built asset.

The interview process provided us with several examples that exhibit the unique nature of both the risks and options inherent in a given real estate project. One such example was a large, mixed-use development that included retail, residential condominiums, office, hotel and parking uses. The key risks associated with the project related to only certain product types. While the project's location presented a strong retail market, the office and residential markets were largely untested. As a result, the project's key risk factors were office rents, condominium prices, and condominium absorption timing.

When the number of uncertainties in a project can be quantified and condensed in a tractable manner, the real options at a developer's disposal can also be narrowed down to those that have the greatest capacity to not only mitigate downside exposure, but capture upside potential. In the above example, the options that the developer was considering in response to the key risk factors included the size and type of the office buildings, the number and type of residential units, and the timing of the phases. In sum, modeling the uncertainties and options in what seems to be an intractable project is easily accomplished when the developer takes the time to isolate the relevant risk factors and options at her disposal.

2.2. Developers Value Real Options Indirectly

Interviewees were asked to describe their current valuation processes for development projects. All interviewees reported using cash flow projections, although the lengths of the projections ranged from a few years to multiple decades. The metrics calculated ranged from simple static ratios, such as the return on cost or cash on cash[5], to multi-period after-tax metrics such as the internal rate of return (IRR) and net present value (NPV). The level of detail in projections and calculations depended on the complexity of the projects, reporting requirements of the project stakeholders, and personal preference of the decision makers.

Many interviewees used static measures to make initial "go / no go" decisions. The most frequently cited measure was the "return on cost" or "development yield," obtained by dividing the projected stabilized net operating income ("NOI") by the total cost of the project. A project is deemed financially compelling if its return on cost is 100 to 300 basis points over current cap rates for existing property of comparable quality in the same location. The required spread is often adjusted subjectively based on each project's perceived risks and upside opportunities.

Many developers focus on the development yield because they view projects much like constant-growth perpetuities. Once stabilized, the built assets are not expected to change significantly in terms of how much income they generate over the foreseeable future, and developers can predict the growth in cash flows more easily by using a single long term growth rate that does not change over time. As shown below, the development yield is analogous to the constant growth perpetuity formula, where the net operating income is equal to the initial stabilized cash flow CF_1 in the numerator, and the development yield is equal to the discount rate (r) minus the growth rate (g) in the denominator.

Constant Growth Perpetuity: $Value = CF_1 / (r - g)$
Development Yield: $Value = NOI / development\ yield$

---

[5] The development yield is equal to the NOI minus interest expense divided by the equity portion of an investment.

The widespread use of a static measure may be surprising given its simplicity; however the reasons for its use have merit. Though both the static and multi-period valuation methods mentioned in this section do not explicitly model uncertainty and flexibility, decision makers do ultimately evaluate both. They do so in two key ways: by adjusting the values of assumed risk and return and by performing sensitivity analysis for the key uncertain variables associated with a given project.

The required returns and hurdle rates for any given project have some degree of subjectivity. Developers tend to assess each project individually and will adjust their return targets for a project according to the risks they feel are associated with that project; specifically, as uncertainty increases, risk (and required return) increases; and, likewise, as flexibility increases, risk (and required return) decreases. With the myriad of factors that go into a development, there is no mathematical calculation that a developer uses to assign a specific level of risk or a required return to each project. Rather, the only way to make such a calculation is through experience and judgment. In this sense, measuring the risk and return of a project is, as one interviewee remarked, more of an art than a science. McDonald (1998) adds credence to the claim that common financial metrics, or "rules of thumb," including hurdle rates and profitability indices, do in fact incorporate the value of real options. He concludes that rules of thumb generally capture at least 50% of a project's option value, and often as much as 90%. The result of this phenomenon is that using such metrics yields near-optimal investment decisions.[6]

Many interviewees reported performing a sensitivity analysis on key variables as part of their standard financial due diligence. This sensitivity analysis shows the impact that changes in any given variable will have on the baseline return calculations. It is usually geared towards assessing downside risks since developers tend to be more concerned with potential losses than potential gains.

---

[6] Though McDonald also found that a broad range of investment rules gave roughly similar outcomes, the hurdle rate "rule of thumb" tends to be more appropriate for low cash flow, long-lived assets, whereas the profitability index rule works better for higher-risk investments.

To help gauge what these key variables are, we asked interviewees what inputs had the largest impact on returns and were the most difficult to predict. The most common variables cited were rents, construction costs, cap rates, and development timing. The reasons given for each input varied. For instance, construction costs are by far the largest cost portion of the project, and tend to have a lot of variability. In contrast, cap rates have a substantial impact on returns and must be predicted further out in the investment cycle. As discussed, these key variables are likely to change for each individual project based on its unique characteristics.

Given that developers adjust their required returns based on a project's risk (which is a function of uncertainty) and flexibility (which is a function of the real options embedded in a project), while performing sensitivity analysis around key risk factors, it is not unreasonable to conclude that developers value real options indirectly. Although these current methods involve a degree of subjectivity, they account for the value of real options nonetheless.

2.3. Potential Barriers to Real Options Analysis Implementation

Most interviewees expressed interest in a simple and transparent real options valuation tool, but voiced concern regarding the complexity of current real options valuation methodologies. Furthermore, interviewees remarked that the validity of any simulation is highly dependent on the distribution parameters it assumes. Specifically, if the nature of the uncertainty (measured by volatility) that is built into the model does not reflect reality, many developers would mistrust the results. We attempted to take into account these factors when developing our model as described in Chapters 5 and 6.

3. A Brief Background on Real Options

Real options analysis provides a framework for analyzing flexibility in development projects by taking into account a manager's ability to react to uncertainty. Developers are aware of the many risks and uncertainties in real estate, and the various tools they have to mitigate them. They are also aware that traditional discount cash flow analysis does not directly account for the value of flexibility. This chapter begins with a brief introduction to traditional options, along with key definitions, and the application of real options theory to real estate development.

3.1. Real Options Terminology

Real options are markedly different from financial options in that their value is based on a physical asset rather than a security such as a stock. Nevertheless, real options and financial options share some terminology, which is worth noting here:

- A call option is the right but not the obligation to purchase an underlying asset for a predetermined price (the "strike" price).
- A put option is the right but not the obligation to sell an underlying asset for a predetermined strike price.
- An American option can be exercised on or before its maturity date.
- A European option can only be exercised on its maturity date.[7]
- A compound option is an option on an option.
- A rainbow option is any option that is exposed to more than one source of uncertainty.

A call option is said to be "in the money" if the value of the underlying asset is above the predetermined strike price; the converse is an "out of the money" option. If an option is "in the money", there is a positive payout. If it is "out of the money", there is no point in

---

[7] Real options can be perpetual in nature; a common example is land ownership, which is described in section 3.2

immediate exercise. Regardless, one cannot lose money on the option after it is purchased (obviously, the cost of the option itself is sunk).

The value of an option is equal to its intrinsic value (the value if exercised today) plus its time value (a.k.a. "option premium"). Not to be confused with the time value of money, the time value of an option reflects the possibility that the option may increase in value due to movements in the price of the underlying asset. Hence, option value is sensitive not only to the current value of the underlying asset and the predetermined strike price, but also the volatility of the underlying asset, the time to expiration, the underlying asset payout rate, and the interest rate.

A familiar example that helps clarify option terminology is a land option. A land option is a type of call option that is commonly used in real estate. Though it can be structured in numerous ways, the basic land option provides a buyer with the right but not the obligation to purchase a piece of land at a given price (the strike price). The strike price is normally equal to the residual land value calculated for the proposed project. If the value of the land is equal to or greater than the strike price, the option is "in the money" and the developer will purchase the land. If not, the option is "out of the money" and the developer can walk away. The main value of the option is that it reduces the risk of the investment by providing the developer with time to gain more knowledge, thereby reducing uncertainty. The value of the option is positively correlated with the length of the option and the amount of uncertainty associated with the project.

The two most common methods of valuing financial options are the Black-Scholes model and the binomial option pricing model. The Nobel Prize-winning Black-Scholes model, presented by Black and Scholes in 1973, is comprised of a system of equations that can be used to value a European call option on a non-dividend paying asset. The model can be derived in various ways, but the most traditional method is based on constructing a riskless hedge portfolio that replicates the returns of holding the option. In 1979, Cox, Ross and Rubinstein published the binomial option pricing model. It is based on constructing a binomial tree of possible future stock prices. Once again, by using the theory of constructing a replicating portfolio, one can value the option at each "node" of the tree, and

work backward to arrive at the value of the option. The binomial option pricing model allows valuation of American options on dividend-paying assets.

3.2. Real Options in Real Estate

The main difference between real options and financial options is that the underlying asset for real options is physical rather than financial. Further, real options models incorporate a manager's ability to adapt to changing conditions, thereby representing the option holder's ability to directly influence the value of the underlying asset.

Any course of action that a manager can take to adjust to uncertainty or mitigate risk is a real option, as long as it involves no pre-commitment or necessary downside exposure. When one looks closely at real estate development, it becomes evident that numerous real options exist. Following is a list of real options that are common to real estate:

- The option to defer or expand a project (a call)
- The option to abandon a project or scale the project back by selling a fraction of it (a put)
- An option to extend the life of a project (a call)
- An option to phase a project (a compound call)
- An option to switch between different modes of operation (a portfolio of puts and calls)

Much of the prior research in applying real options analysis ("ROA") to development focuses on phased development (a compound call) and simple land ownership (an American call), which is described in section 3.3. A phasing option occurs when a developer can build an initial phase of a project and respond to its performance in designing future phases. For instance, for large, multi-building developments, it does not always make sense to commit fully to the construction schedule at time zero. Rather, it reduces risk to divide the project into separate phases. The first phase may be committed to at time zero, but the developer maintains the flexibility to delay or cancel additional phases if they do not make economic

sense in the future. A phasing option can be either compound or parallel. In the case of the former, exercising one option by building a phase presents another option to build a subsequent phase. The latter refers to phases that are independent of one another and can be built at the same time.

Though phasing and land ownership have commonly been discussed in academic literature, numerous additional examples of real options in real estate become apparent when one considers instances in which there exists a right but not an obligation to take a course of action based on new information. The following are just a few of these examples:

- Developers often have the option to change the intended use of a project (a call option). For instance, consider an office building that can be converted to residential. Here the underlying asset is the value of the proposed residential project with the value of the office building (plus the construction or conversion cost) as the strike price. Another example would be an apartment building that can be converted to condominiums.
- The option to expand or contract applies to many development projects. Common examples are scaling a residential building down in size in order to save costs (often by switching from concrete or steel construction to a wood frame building).
- Options can also be applied to project financing. For instance, the equity provider in a levered real estate project has a call option on the project with a strike price equivalent to the outstanding value of debt. In other words, if the value of the property is above the value of the debt, it is "in the money" and the equity provider has positive value of his equity.

3.3. Land as a Call Option and the Samuelson-McKean Formula

The call option model of land value provides a framework by which we can link the values of the underlying land with the timing and value of real estate development. Land gives its owner the right but not the obligation to develop property at any point in the future. Hence, land can be valued as a perpetual call option for which the payout is equal to the difference between the present value of a built asset and the present value of the cost to build it, excluding the cost to acquire the land. In the call option model of land value, the built property represents the underlying asset and the construction cost represents the strike price of the option.

The Samuelson-McKean (S-M) formula is a system of equations used for solving a perpetual call option in continuous time, and is a convenient and simple method for finding an approximate solution to land value. The S-M formula is based on the same assumptions of economic arbitrage as Black-Scholes, however Black-Scholes values European options (or American call options without dividends) that have a finite rather than perpetual maturity. Like the Black-Scholes model, the S-M model is based on a number of simplifying assumptions[8], including the following:

- Real estate markets are highly efficient, or "frictionless". Alternatively, in the absence of an efficient market, the model can be viewed as providing a normative valuation for the option.
- The market value of the underlying asset exhibits a random walk in time around a constant growth rate.
- The returns on the underlying asset are normally distributed.
- Construction costs are riskless and grow at a constant rate over time.[9]

---

[8] Some of these assumptions can be relaxed using more sophisticated models, such as Childs, Ott, and Riddiough (2002). However, the S-M formula generally seems to agree with empirical reality.

[9] The model can easily reflect stochastic construction costs by a simple transformation in which the construction cost is taken as a numeraire and the option value is expressed per dollar of construction cost. See Appendix 27 in Geltner and Miller (2007).

Despite its simplifying assumptions, the Samuelson-McKean formula is a powerful tool that can be used for obtaining valuable insights on land value as a function of the then-applicable market conditions.

The following inputs are needed for the Samuelson-McKean Formula:

- Current value of the underlying asset (V_0)
- Cost to build the asset, excluding land (K)
- Volatility of the underlying asset returns ( )
- Dividend yield (y_v), approximated by the cap rate
- Risk free rate (r_f)
- Construction cost yield (y_k), approximated by the risk free rate minus the growth in construction costs

Given the above inputs, the option elasticity   can be determined:

$$\eta = (y_v - y_k - \sigma_v^2/2 + \left[\left(y_k - y_v - \sigma_v^2/2 \right)^2 + 2y_k\sigma_v^2 \right]^{\frac{1}{2}})/\sigma_v^2$$

The option elasticity is then used to determine the optimal development hurdle value V^* (1) and the value of the land as of time zero (2):

$$V^* = K \times \eta/(\eta - 1) \qquad (1)$$

$$C_0 = (V^* - K_0) \times \left(\frac{V_0}{V^*} \right)^{\eta} \qquad (2)$$

The real options model presented in this thesis uses the Samuelson-McKean formula in the following capacities:

- The hurdle value V^* is used to decide whether or not to exercise the option and proceed with a development at any given point in time.[10] If the present value of the built asset is below the hurdle value, then land should be held undeveloped; if it is above the hurdle value, land should be developed immediately.

- The abandonment value of selling the land if we do not develop it within the given time horizon is calculated using equation (2).

Though we are able to calculate the opportunity cost of capital for the "live option" period alone using the Samuelson-McKean formula, we do not use it in the model. Rather, for simplification, we use a single opportunity cost of capital, which can be approximated by the developer's hurdle rate or desired IRR.

---

[10] For any given project, using conventional methods, if at time t we had a positive NPV, we might be tempted to proceed with the development. Although it might make intuitive sense, this approach may not maximize the residual land value. Since land represents a call option, we must be consistent with Option Valuation Theory in choosing when we exercise our option by developing on the land. The value of any option equals its intrinsic value (the NPV of the development project if started at time t) and its time value, which captures the possibility that the option's payoff might increase in the future. If we develop whenever the NPV is greater than zero, we may be exercising our option prematurely and losing time value. However, if the value of built property exceeds V^*, there is no time value left and development is optimal.

4. Prior Work in Applying Real Options to Real Estate

The bulk of prior work in applying real options analysis (ROA) to real estate development projects is based on traditional economic option valuation theory (OVT). Yet, OVT-based models that have been applied to real estate projects have been slow to catch on in the mainstream development community. Practitioners have found the models overly complicated and confusing. In response, the engineering community developed a more pragmatic options model based on Monte Carlo simulations in Excel. The strengths and weaknesses of both approaches are discussed in this chapter.

4.1. Option Valuation Theory

Various studies have applied traditional option pricing models to empirical data such as land transactions (Quigg, 1993), as well as general property transactions (Sing and Patel, 2001). In addition, Espinoza and Luccioni (2005) apply the Samuelson-McKean formula to brownfield remediation projects. Yet, there are few papers that have apply OVT directly to multi-phased real estate development projects. Kang (2004) applies decision tree analysis and real options valuation in case study format to a large private development in Hong Kong. Further, Hengels (2005) presents an Excel-based financial model for valuing complex projects.

The most recent studies that have applied ROA to real estate development projects, including Kang and Hengels, have relied on the binomial option pricing model. Binomial trees are not commonly used in the development field. Further, they can be difficult to learn, especially for complicated options scenarios. Thus, the use of binomial trees presents a barrier to the immediate adoption of real options analysis in the real estate field.

4.2. The Engineering Approach

In response to the limitations of OVT, leaders in the engineering and decision sciences fields developed a more hands-on approach to evaluate real options based on Monte Carlo simulations in Excel. This engineering approach has already been applied to a number of engineering-related fields, including manufacturing, infrastructure development, and natural resource extraction. De Neufville, Scholes, and Wang (2005) applied this methodology to real estate using a parking garage example. Cardin (2007) took the application to real estate one step further by using a large, phased apartment project as a case study.

The engineering approach has many advantages over the financial approach to valuing options when considering applications to development projects. The engineering approach is more transparent in that it is based on a traditional discounted cash flow model, and can reflect a firm's particular decision rules and policies. It also does not require the user to learn new financial concepts and methodologies, and can model more than one source of uncertainty transparently. Further, it introduces the concept of an "expected net present value" (ENPV) and plots a cumulative distribution function of the possible outcomes via a Value at Risk and Gain ("VARG") curve. This probabilistic representation of outcomes allows developers to better understand the risk associated with a project, as well as the magnitude of potential downside and upside scenarios.

An in-depth study of the engineering approach has been provided in Cardin (2007). This approach is based on creating a catalog of operating plans that specifies specific design elements and decision rules for a project under a limited set of uncertain future scenarios; for instance high, medium, and low growth in demand. In order to create the catalog of operating plans, one must examine most of the different pre-specified combinations of decision rules and design elements. The method by which Cardin does this is called adaptive One Factor at a Time (OFAT). The catalogue specifies the best way the project should be designed and operated among a limited set of alternatives under the different pre-specified uncertain scenarios. The idea is that designers and program managers will know the best way (out of a limited range of possibilities) to design and operate the system to increase its performance given the market scenario they feel will unfold in the future. Once the catalog

of operating plans is created, Monte Carlo simulations are run to assess the value of a deterministic system versus one that incorporates flexibility via the catalogue.

Despite its value for complex engineering design problems, the adaptive OFAT procedure may not be necessary for development projects. As discussed in Chapter 2, developers are often able to narrow down the flexible options that are associated with a project. In addition, there are usually standard pre-specified decision rules that developers use to evaluate projects, the bulk of which are loosely based on the standard financial NPV rule. Thus, a search among different flexible options is not necessary in most cases. In addition, the adaptive OFAT process can be time-consuming and must be customized to each project. Despite the value of adaptive OFAT for complicated engineering projects, the use of adaptive OFAT, or other methods of searching the combinatorial space, may not be necessary for development projects and may present a barrier to widespread ROA implementation in the short-term.

4.3. The Solution: A Hybrid Model

The model that we develop in this thesis is a hybrid of both the traditional options valuation approach and the engineering approach. We combine elements from both methodologies in order to achieve the model's goals. These goals include:

- Making the model as simple and straightforward as possible;
- Maintaining as much theoretical integrity as possible in applying real options analysis;
- Designing the model so that it dovetails with the methods by which developers currently evaluate projects;
- Incorporating the decision rules commonly used in development; and
- Making the inner workings of the model transparent so that it can be modified or expanded upon for multiple situations.

The model is based on a discounted cash flow approach, which most developers are already familiar with. Further, it uses Monte Carlo simulations to reflect the uncertainty that exists in real estate *pro forma* variables.

Apart from this basic structure, the remainder of the model is based on the traditional economics approach. Though some of the concepts we introduce will likely be new to the development community, the bulk are commonly used and understood. The new concepts that we introduce are based on the Samuelson-McKean formula discussed in section 3.2. As the main decision rule, we have adopted the basic NPV criteria for which the appropriate discount (or hurdle) rate can be entered by the practitioner. We use this decision rule in conjunction with the S-M hurdle rate (as described in Section 3.3) as a criterion for starting development. In addition, we use S-M to calculate an abandonment value of land if the option is not exercised. Though we have entered the discount rate as an input field in the model, the S-M formula also provides insight on what the appropriate opportunity cost of capital should be before the option is exercised (the speculative land discount rate).

5. The Real Options Model

Taking into account the findings of our literature review and industry interviews, we developed a valuation model that provides a "proof of concept" for real options analysis. The model is applied to the following commonly-faced scenarios in real estate development:

- When and how to phase a project instead of building it all at once
- The valuation of various development programs under uncertainty
- Valuation of the option to defer a development project in suboptimal market conditions

Microsoft Excel ® software was used to create the model, with a focus on transparency and simplicity. Considering that modeling all of the uncertainties and embedded options in a development project would be a nearly impossible task, we simplified the model to address specific real options scenarios, thus creating something practical and easy to use. The model can easily be expanded or modified, and serves as a framework for developing a customized tool to simulate a variety of situations.

5.1. Model Capabilities and Theoretical Foundation

The model is a hybrid of the financial and engineering approaches to real options analysis. Its projections and decision rules are based on a discounted cash flow methodology and the assumption that developers want to maximize net present value under any given set of market conditions.[11]

The behavior of the underlying asset (the value of the built property) follows a stochastic process based on parameters input by the user. Specifically, the growth in built asset values is dispersed randomly based on a normal distribution curve, while cost follows a

---

[11] Although this assumption is applied in the model, the model's output does not reflect the global maximum NPV of the project, but instead serves as a tool for making strategic decisions during its design and implementation.

deterministic linear growth pattern. The starting value of the built asset also varies randomly based on a specified uncertainty factor. **Figure 1** depicts three sample projections of built property value.
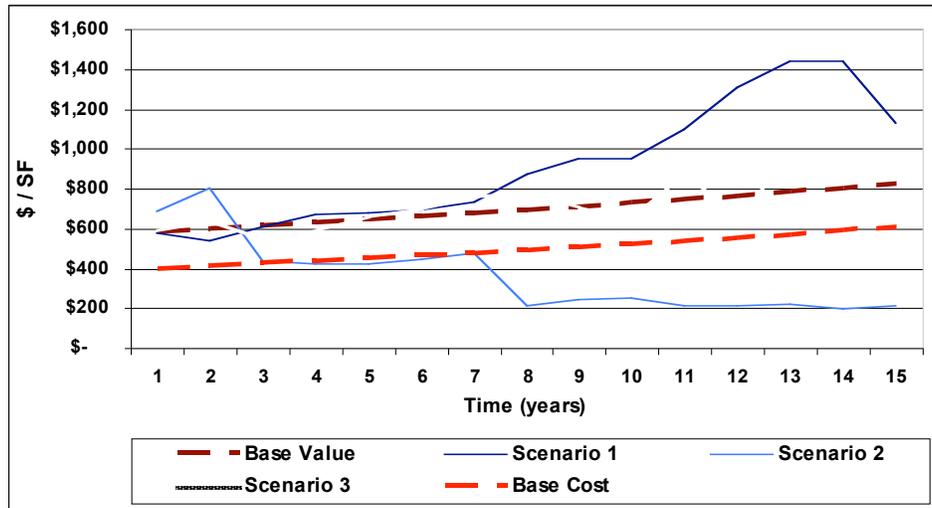
Figure 1: Three potential random distributions. The green line is built asset value grown at 2.5% per year. The red line reflects costs grown at 3.0% per year. Scenarios 1, 2, and 3 are possible randomly-generated built asset value growth scenarios, with 15% volatility.

5.2. Modes of Operation

There are two main modes of operation for the model, depending on the scenario the user wishes to evaluate. The user inputs key assumptions about two development programs for a subject site. The inputs for each program include specifications such as square footage, built value (as of the present time, determined by dividing NOI by the cap rate), cost (excluding land), timing (construction and lease-up), development project discount rate, cap rate, uncertainty of the built asset's initial value, and volatility around built asset value growth. The model then projects built asset value (which is dispersed randomly) and cost (which grows deterministically). At each year, *t,* the model follows a decision rule as described below.

- In the *Switch*[13] mode, the model treats each development program as mutually exclusive. At any given year, t, the model chooses to develop the use with the highest projected NPV, assuming one or both of the programs' built asset values exceed their respective hurdle values (as described in section 3.3). If both programs are NPV negative, the model defers development until the following year for a maximum of nine years. If development is still not feasible by the end of year nine, the model calculates a residual land value for both uses (by way of the Samuelson-McKean formula), and captures the higher of the two values.

- In the *Phase* mode, the model treats both programs as part of a phased development in which the first and second programs are to be built in their respective order. The first phase is developed when necessary criteria are met (as described in section 5.3.2), and the second phase starts no earlier than one year after the first phase, and only if it meets the same set of criteria. Residual abandonment value (if the option to develop is not exercised by the end of year nine) is calculated using the Samuelson-McKean formula. In the case where only the first phase is developed, a partial land sale is assumed.

In both modes, the model includes the value of the option to defer development for up to nine years unless the user instructs it to force development in time zero[14].

---

[13] The *Switch* mode is not to be confused with a switching option. Most often, multiple permitted uses on a site reflect an American call option on the greater of two or more assets.

[14] This function is used to simulate a world without flexibility.

5.3. Model Inputs and Operation

In both modes, the model takes as inputs specifications for two development programs. For each program, the user specifies:

- Net rentable square footage ("NRSF")
- Development cost K_{Dev} per net rentable square foot (excluding the cost of land and permitting) and annual growth in costs (g_{KDev})
- Net operating income ("NOI") per square foot
- Projected annual growth in built asset value (g_V)
- Capitalization rate used to determine current built asset value per square foot (V)
- Initial NOI uncertainty factor (ϵ_1)
- Time to permit (t_P), construct (t_C), and lease up the asset (t_L), with a maximum of one, three, and two years, respectively
- Annual volatility in built asset value (ϵ_2)
- Project discount rate (r)[15]
- Risk free rate (r_f)

The *Switch* mode has some additional inputs. A "weight factor" input determines how closely the stochastic processes for the two programs are correlated (in the *Phase* mode the model implicitly sets this value to 1, implying that both phases are perfectly correlated since they are assumed to be of the same product type). In addition, for each program, the switching cost per net rentable square foot (K_{Switch}), annual growth in switching cost ($g_{KSwitch}$), and time to switch are used to model the process of rezoning or re-entitling the site for a new use.

---

[15] This discount rate reflects a development opportunity cost of capital (OCC) rather than the OCC for a stabilized asset. Hence, it can be used to discount cash inflows and outflows.

5.3.1. Projection of Value and Cost

Once the necessary inputs have been entered, the model makes the following projections for each development program for 16 years:

- At time zero, the built asset value V is determined by dividing the projected NOI by the cap rate, and dispersing the resulting value using a normal distribution function with a standard deviation equal to $_1$.

- Each year, V is grown by a factor using a normal distribution function with mean $1+g_V$ and standard deviation $_2$. Development costs and switching costs (if applicable) are grown at their input rates.

- In the *Switch* mode, there are two randomly generated growth rates for the built asset. The user can specify random values of one use as a function of the other. At any given time t, the randomly generated growth rate for the first program is x. There is a second randomly generated growth rate, y, and the user inputs the value a (the "weight factor") such that the growth rate for the second program equals $(a \times x) + (1-a) \times y$. This parameter serves as a proxy for modeling correlation between the built asset values for different product types.

5.3.2. The Decision Making Process

At any given year t, the model uses a discounted cash flow (DCF) process to project the then-current value and costs for each program as follows:

- The then current built asset value V is grown at a rate of g_V for a period of T_{Total} years, where T_{Total} is the sum of the switching (if applicable), construction, and lease-up times. This reflects growing the built asset value at the user-input growth rate until the time of stabilization. We will call this value V_s. To obtain the present value of V (PV (V)) we take V_s and discount it back T_{Total} years, using the development project discount rate r.

- We obtain the present value of the cost to build the asset $PV(K)$ (where K is the sum of construction and switching cost) in the same fashion, using their assumed growth rates to project current values forward, and using the project discount rate r to discount them back to present time.

- The NPV of the immediate exercise of the project (excluding land cost) at time t is defined as $PV(V) - PV(K)$.

In addition to calculating the project NPV, the model calculates the threshold value V^* for which it would be optimal to develop at time t.[16] For either program at time t, if the present value of the built asset exceeds V^* (which will only happen in positive NPV cases), the model makes a decision:

- In the *Switch* mode, the model makes a "go" decision to develop the program. In the case when both programs exceed their hurdle values, the model chooses the program with the highest NPV. Once a development program is chosen, the other program is abandoned.

- In the *Phase* mode, a single phase of the multiphase program will be developed. The second phase can be developed no earlier than one year after the decision is made to develop the first phase.

Once a decision is made to develop, cash outflows are realized for the switching (if applicable) and construction phases in the subsequent years. In the case of multi-year construction, costs are spread evenly over the construction time period. Cash flows during the lease-up period are not accounted for in this model due to their low projected impact on total project value.[17] At the end of the lease-up period (or construction period if no additional lease-up time is assumed), the built value of the project is realized, and the model captures market value as if the project were sold.

---

[16] Explained in section 3.3
[17] Simplifying assumptions of the model are discussed in a later section.

All of the project cash flows are captured in the "net cash flow" line and discounted back to time zero using the project discount rate r. If the decision to develop is not made by the end of year nine, the model assumes abandonment at the following value:

- In the *Switch* mode, the higher of two abandonment land values (as determined by the Samuelson-McKean formula) is realized.
- In the *Phase* mode, if only one phase is developed, a partial land sale is assumed.

5.3.3. Monte Carlo Simulation

The aforementioned discounting and decision-making processes are iterated 5,000 times to reflect a variety of possible outcomes. The following statistics are presented:

- The expected NPV
- Standard deviation of NPVs
- Minimum and maximum NPVs
- Median NPV
- Percentage of trials that result in a negative NPV
- Percentage of trials in which development never occurs

5.4. Interpreting Model Results for Common Scenarios

Now that we have covered the model framework, it is important to discuss how to understand its output and use it to make strategic decisions when creating a development program or acquiring a land site. Here are some common applications:

- *Determining when to phase a project versus building it all at once* – in this scenario we want to use the *Phase* mode, and input the parameters for phases one and two of the project. The user can force the model to operate in "Static" mode and proceed with the development at time zero in all scenarios to get a base case value of the phased project without flexibility. Putting the model back into the normal mode reveals the

project's value with flexibility. The user can then input the values of a single phase project (and zero out the inputs of the second phase) to get the base case value with and without flexibility. The user can then compare the value of both the flexible and static cases for the single building and phased building programs to determine the best course of action.[18]

- *Determining the added value of multiple permitted uses* – in this scenario we want to use the *Switch* mode, and input the parameters for a single permitted use. After the output has been noted, we input the parameters of the second development program. The model tells us how much value is added by being able to choose among uses, in addition to what percentage of the trials involves developing one program as opposed to the other.

- *Determining the value of being able to delay a project for up to nine years* – this value is implicitly calculated in both modes as the model automatically defers development for up to nine years if market conditions are unfavorable.

In an industry setting, the outputs of this model will help more accurately determine the ideal development program by providing a range of possible NPV realizations. Though the model's output should not be used to directly derive land value, an approximation of the difference in value between programs can serve as a proxy for the highest and best use.

---

[18] This comparison is presented in the case study in Chapter 6.

5.5. Assumptions and Limitations of the Model

The model makes some universal assumptions that are worth noting here:

- Cash flows are realized and "go/no go" decisions are made at the end of each year.
- The time in which the developer can choose to exercise his option to develop is limited to nine years (the last possible chance to make a decision is at the end of year nine), after which the model assumes abandonment by capturing the then applicable land value.
- Cash flows during the lease-up period are not accounted for in the model. Given the difficulty in ascertaining these cash flows, and their minimal projected impact on value, the effect of this assumption is negligible.
- Construction outflows are evenly distributed over the construction period.
- In the *Switch* mode, there is no switching hurdle above and beyond the Samuelson-McKean hurdle rate (V^*). According to Geltner, Riddiough, and Stojanovic (1996), the right to choose among multiple uses adds up to 40% to land value, thus creating an additional hurdle value to be crossed before development is optimum. We ignore this extra premium for the sake of simplicity.
- The developer cannot abandon the project once the decision has been made to develop.
- Construction cost risk is not implemented, although the model can easily be modified to account for this risk.
- It is assumed that discount rates for the project stay constant throughout the entire year period.
- The model handles uncertainties using a normal distribution function. Some debate exists in industry as to whether a "fat tail" distribution may be more appropriate. Although normal distribution serves as a good proxy for uncertainty, the model can be modified to allow for other distributions, including lognormal or triangular.[19]

5.6. Practical Note for Use of the Model

---

[19] In fact, almost any feature of the model can easily be changed with minimal reprogramming.

To minimize calculation time, it is highly recommended that Excel be run in manual calculation mode when using the model, and that no other models are kept open. We have found that each simulation may take up to one minute to complete.

6. Case Study: The Value of Phasing

Now that we have gone over the theoretical aspects of the model, it is time to see how it works. The following case study is meant to illustrate the use of the model for real world situations, and the interpretation of its results. The case is based on an actual development project.

6.1. Case Background and Assumptions

A west coast developer was considering developing a suburban office project. Demand for office product in the area was uncertain. However the site was large enough that she could build the project in two smaller, phased buildings instead of one large building while achieving the same net rentable square footage under allowable zoning. She wanted to know whether the single building or the phased building program yielded a higher net present value. **Table 1** provides a summary of the two programs.

| | Single Phase Project | Two-Phase Project | |
		Phase I	Phase II
NRSF	350,000	175,000	175,000
Total Development Cost / NRSF	$ 400.00	$ 445.00	$ 385.00
Cost growth	3.0%	3.0%	3.0%
NOI / NRSF	$ 32.00	$ 32.00	$ 32.00
NOI growth (annual)	2.5%	2.5%	2.5%
Construction Period (years)	2	2	2
Lease-Up Period (years)[20]	2	1	1

Table 1: Case study assumptions

---

[20] Given the larger size of the single phase project, it would take an additional year to lease up after completion of construction.

In addition, we have the following assumptions that are applicable to both programs:

- Cap rates for office product are projected at 5.5%, resulting in a per square foot value of approximately $582.

- The project discount rate is 15%. This single discount rate represents a weighted average of all of the phases of the development cycle and is used to discount each cash flow back to time zero.[21]

- The initial NOI is estimated to deviate up to 10% from the projection.

- Built property is estimated as having a standard deviation of 15%, per Geltner and Miller (2007).

- The risk free rate, approximated by the ten year treasury, is 5%.

6.2. Options Evaluated in this Case Study

There are multiple scenarios that we can evaluate using the model to get a more accurate idea of which scheme maximizes the ENPV of the site:

- **Static Case with No Options:** The ENPV of each program without the option to delay or abandon development. In other words, both the single phase building and the first phase of the two-phased project would start at time zero, and the second phase of the two-phased project would start at the end of year one.

- **Deferral Option:** The ENPV of each program with the option to delay development for up to nine years. This takes into account the option that the developer may delay development should market conditions turn out worse than expected prior to making the decision to start construction. As mentioned in Chapter 5, this option expires after the end of year nine when the model assumes that the land will be sold for its then-applicable value.

---

[21] Though each phase of the development is characterized by a different risk and return profile, we used one single blended discount rate for simplicity.

- **Value of the Option to Phase:** the option to build the project in phases as opposed to all at once is determined by subtracting the ENPV of the flexible single building program from the ENPV of the phased building program.

In the following section we outline how to use the model to value the above options. It is not necessary to run all of these steps when using the model; we are doing so to demonstrate the various ways in which the model can be used, as well as how to interpret its output. Below is an outline of the process:

1. We begin by valuing the project without flexibility and without uncertainty. This is accomplished by way of a simple NPV calculation for the single and phased building programs. This step serves to provide a basis for comparison, and does not require use of the model.

2. After calculating the deterministic NPV, we use the real options model to add uncertainty around our initial projections of NOI, as well as volatility in the subsequent growth in built asset value. As we are not yet valuing flexibility, we instruct the model to start development immediately. This step is run twice; once for the single building program and once for the phased building program. In the single building program, the model will always make the decision to build at time zero; in the phased building program the model makes the decision to build the first phase at time zero and the second phase at the end of year one.

3. Once we have the expected NPV for the static case with uncertainty for both programs, we run the model again, this time allowing it to delay development for up to nine years (and sell the land if development never occurs). We also conduct this step twice to obtain results for the single and phased building programs.

After the above three steps are run, we examine the various data regarding the expected NPV, its standard deviation, and its distribution. This information will allow us to approximate the value of flexibility for each program, and ultimately compare the two programs to select the best use for the site:

- The value of the option to wait is the difference between the ENPV with and without the option to wait for each respective program.
- The value of the phasing option is the difference between the ENPV of the two programs.
- The highest and best use of the site is the program that yields the highest ENPV.

6.2.1. Step 1 – Finding the Deterministic NPV

This step does not require use of the model. It is a mere baseline for comparison and a reflection of current valuation methodologies. We calculate the NPV of the single and phased building programs given the assumptions in **Table 1**. These calculations are included in **Figure 2** and **Figure 3**.

	Time 0		Year 1		Year 2		Year 3		Year 4	
Built Asset Value per SF	$	582	$	596	$	611	$	627	$	642
Cost Per SF	$	400	$	412	$	424	$	437	$	450
Timing	**Decision**		**Construction 1**		**Construction 2**		**Lease-Up 1**		**Lease-Up 2**	
Total Cost			$	(72,100)	$	(74,263)				
Total Built Asset Value									$	224,776
Cash Flows	$	-	$	(72,100)	$	(74,263)	$	-	$	224,776
NPV	$	**9,668**								

Figure 2: Deterministic NPV for Single Building

	Time 0		Year 1		Year 2		Year 3		Year 4	
Built Asset Value per SF	$	582	$	596	$	611	$	627	$	642
Cost per SF - Phase I	$	445	$	458	$	472	$	486	$	501
Cost per SF - Phase II	$	385	$	397	$	408	$	421	$	433
Timing Phase I	**Decision**		**Construction 1**		**Construction 2**		**Lease-Up**			
Timing Phase II			**Decision**		**Construction 1**		**Construction 2**		**Lease-Up**	
Cash Flows - Phase I			(40,106)		(41,309)		109,647			
Cash Flows - Phase II					(35,739)		(36,811)		112,388	
Cash Flows - Total	$	-	$	(40,106)	$	(77,048)	$	72,836	$	112,388
NPV - Phase I		**$5,985**								
NPV - Phase II		**$13,030**								
NPV - Both		**$19,015**								

Figure 3: Deterministic Pro Forma for Phased Buildings

In each diagram, the *Timing* row explains the various phases of the development.

- *Decision* refers to the period in which the decision is made to build.
- *Construction 1* and *Construction 2* refer to the first and second years of construction, respectively.
- *Lease-Up 1* and *Lease-Up 2* refer to the first and second years of lease-up, respectively (in the phased program there is only one year of lease-up).
- At the end of the final lease-up year, the asset value is captured.

An opportunity cost of capital of 15% is used for both NPV calculations. The calculations show that the NPV of the phased program is substantially higher than that of the single building program – a difference of approximately $9.3 million. This at first may seem counterintuitive, as both of the programs yield the same net rentable square footage and the average per square foot construction cost is slightly higher for the phased program. To understand why the NPV is higher for the phased program, we examine the timing of the cash flows as presented in **Figure 4**. The cash flows represented are discounted to reflect their values as of time zero.
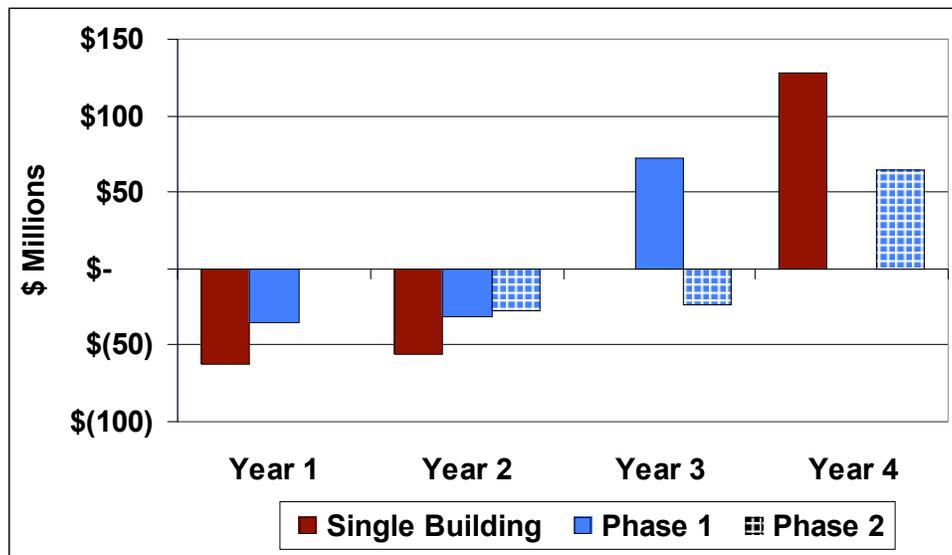

Figure 4: Discounted cash flow diagram for single and phased building programs

The reason that the NPV is higher for the phased building program is that the additional year of lease-up for the single building program leads to a lower present value of the residual amount due to discounting. Also worth nothing is the substantially higher NPV for the second phase of the phased program due to its lower construction costs.

Based on this calculation, the phased building program should be chosen for the site. However, a single NPV calculation does not tell the full story of the range of possible outcomes for the different programs. To get a better idea of the potential distributions for the single and phased building programs, we use the real options model.

6.2.2. Step 2 – Finding the Static ENPV under Uncertainty

To introduce uncertainty to the static case, we employ the real options model. The initial built asset value is projected at $582 with a 10% uncertainty. Thus, in about two thirds of the trials, the initial value will be between $524 and $640 (±10%). The annual growth in value is projected at 2.5%, and in approximately two thirds of the trials, the annual growth will be between -12.5% and 17.5% (±15%).

We begin by putting the model into *Phase* mode. We then input the relevant program data for the single building program as presented in **Figure 5** .[22]

---

[22] Input cells have blue font and are outlined. The "Switching Time" and "Switching Cost / SF" inputs should be zeroed out, as they do not apply in this mode. Also, the "Switching Cost OCC" and "Switch – Weight Factor Between Uses" can be ignored for now.

Scenario: Phase Option (Static Case)									
Option Type	**Phase**								

Assumptions - Today's Values

Use	NRSF	Cost / RSF	Cost Grwth	NOI / RSF	Value / SF	Switching Time	Constr. Time	Leaseup Time	Yr. Stab.
Single Bldg.	350	400.00	3.00%	32.00	$ 582	0	2	2	4
	0	385.00	3.00%	0.00	$ -	0	2	1	3

	Switch Cost / SF	Switch Cost Gr.	Built Value Gr.	Discount Rate	Cap Rate	NOI Uncertainty	Built Val. Vol.	NOI Fluctuation	Const. Cost Yld
Single Bldg.	0.0	0.0%	2.5%	15.0%	5.5%	10.0%	15.0%	85%	2.00%
	0.0	3.0%	2.5%	15.0%	5.5%	10.0%	15.0%	85%	2.00%

Switching Cost OCC	15.00%
Risk Free Rate	5.00%
Switch - Weight Factor Between Uses	25.00%
Correlation	100.0%
Covariance	1.53%
Static Case?	1

Figure 5: Data inputs for single building program

Note that to reflect a single building program, we simply zero out the "NRSF" and "NOI / RSF" inputs for the second program. To determine the static case with no options, we enter a "1" for the input marked "Static Case?". This forces the model to begin development immediately. After running the simulation, the model outputs the expected NPV ("ENPV") and it's minimum, maximum, and median values; the standard deviation of the ENPV ("STDEV"); the percentage of the trials that result in a negative NPV ("% Negative"); and the percentage of the trials in which the decision was made to develop ("% Developed"). In addition, a histogram and Value at Risk and Gain ("VARG") curve is outputted in the "Simulation" worksheet in the model. We note the results of this simulation before repeating this step for the phased program.

Repeating Step 2 for the phased program, we enter the inputs for the program as described in **Figure 6**.

Scenario: Phase Option (Static Case)

Option Type: **Phase**

Assumptions - Today's Values

Use	NRSF	Cost / RSF	Cost Grwth	NOI / RSF	Value / SF	Switching Time	Constr. Time	Leaseup Time	Yr. Stab.
Phase 1	175	445.00	3.00%	32.00	$ 582	0	2	1	3
Phase 2	175	385.00	3.00%	32.00	$ 582	0	2	1	3

	Switch Cost / SF	Switch Cost Gr.	Built Value Gr.	Discount Rate	Cap Rate	NOI Uncertainty	Built Val. Vol.	NOI Fluctuation	Const. Cost Yld
Phase 1	0.00	0.0%	2.5%	15.0%	5.5%	10.0%	15.0%	109%	2.00%
Phase 2	0.00	0.0%	2.5%	15.0%	5.5%	10.0%	15.0%	109%	2.00%

Switching Cost OCC	15.00%
Risk Free Rate	5.00%
Switch - Weight Factor Between Uses	25.00%
Correlation	100.0%
Covariance	3.04%
Static Case?	1

Figure 6: Data inputs for phased building program

6.2.2.1. Tabulation and Interpretation of Results

We have tabulated the results from the simulations run in step 2 in **Table 2**.

	Single	Phase I	Phase II	Both Phases	Delta
ENPV	**9,475**	6,064	13,254	**19,319**	**9,843**
Median	**4,383**	4,071	10,520	**14,617**	**10,234**
Standard Deviation	**41,048**	20,423	20,708	**39,865**	**-1,183**
Minimum	**-89,576**	-46,393	-35,952	**-82,345**	**7,230**
Maximum	**225,074**	105,346	119,675	**215,143**	**-9,931**
% Negative	**44.7%**	41.5%	27.8%	**33.4%**	**-11.4%**
% Developed*	**100%**	100%	100%	**100%**	**0%**
*By Definition					

Table 2: Results of Static Case

As expected, the expected NPVs for both programs are similar to the deterministic NPVs from Step 1. However, the expected NPV does not tell the full story. The standard deviation for the single building program is higher, its minimum NPV is lower, and there are a higher percentage of negative NPV cases. The single building case has a higher maximum value, but that value is only achieved in a few of the trial runs (representing less than one percent of the total). It would therefore be reasonable to conclude that the single building program is not only less financially compelling, but more risky. A comparison of the histograms and VARG curves for the two programs confirms this theory.

The histograms for the two programs are shown in **Figure 7** and **Figure 8**. Though both histograms look fairly similar, the range for the phased program is smaller. Further, the distribution of NPVs for the phased program is closer to a normal, bell-shaped curve (both histograms show a slight positive skew with an elongated right tail). The histogram for the single building program is more strongly skewed, and so a comparatively higher percentage of outcomes are less than the ENPV.
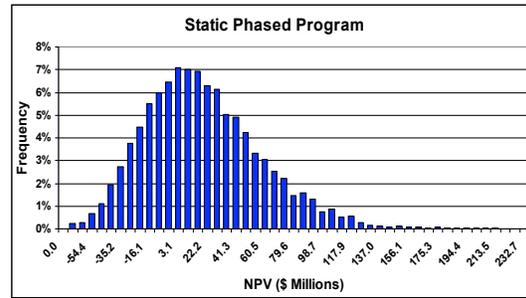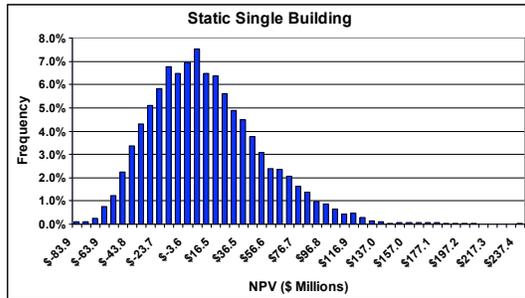

Figure 7: Histogram - Static Single Building Program   Figure 8: Histogram - Static Phased Building Program

In **Figure 9** below, the vertical lines show that the ENPV of the phased building program is greater than the ENPV for the single building. Further, the cumulative distribution function (VARG curve) for the phased building program is shifted to the right of that for the single building program at every point with the exception of top of the curve. This shows that the loss is less and the gain is greater for the phased program than the single building program at every confidence level except for 99% and above, where the maximum value of the single building program exceeds that of the phased building program. For instance, for the single building scenario there is a 10% chance that the developer will lose between $36.5 million and $89.6 million. In contrast, potential losses for the phased building program are between $28.0 million and $82.3 million with the same probability.
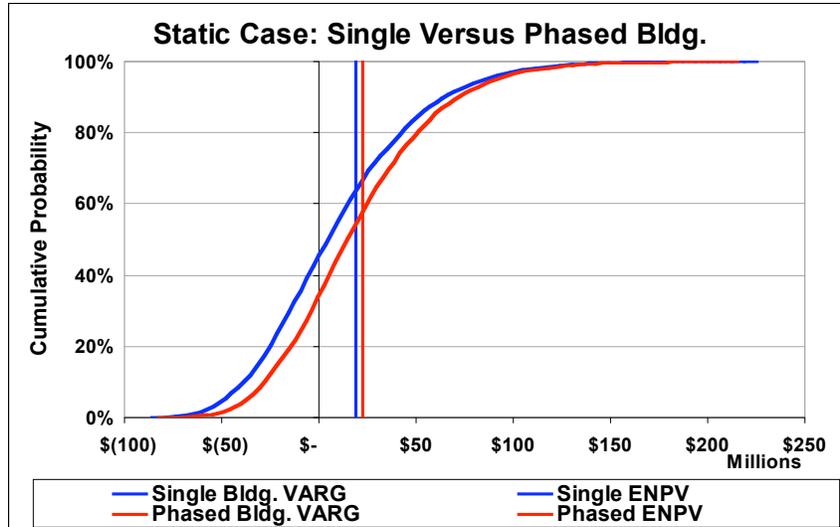
Figure 9: VARG Curves - Single vs. Phased Building

6.2.3. Step 3 – Finding the Flexible ENPV under Uncertainty

To run the model with the option to delay or abandon development, we can repeat the two simulations run in step 2, only we toggle the "Static case?" input to zero before each simulation is run. The results are included in **Table 3**. A side-by-side comparison of the static versus flexible case for both programs is presented in **Table 4**.

	Single	Phase I	Phase II	Both Phases	Delta
ENPV	**16,994**	10,145	12,685	**22,829**	**5,836**
Median	**3,096**	2,289	4,217	**6,804**	**3,708**
Standard Deviation	**29,944**	16,072	17,281	**32,584**	**2,641**
Minimum	**-56,710**	-25,139	-20,730	**-38,483**	**18,226**
Maximum	**251,954**	120,027	117,138	**210,875**	**-41,079**
% Negative	**8.1%**	7.0%	2.8%	**4.8%**	**-3.3%**
% Developed	**48%**	50%	46%	**46%**	**-3%**

Table 3: Results of Flexible Case ($ Thousands)

	Single Building		Phased	
	Static	Flexible	Static	Flexible
ENPV	**9,475**	**16,994**	**19,319**	**22,829**
Median	4,383	3,096	14,617	6,804
Standard Deviation	41,048	29,944	39,865	32,584
Minimum	-89,576	-56,710	-82,345	-38,483
Maximum	225,074	251,954	215,143	210,875
% Negative	44.7%	8.1%	33.4%	4.8%
% Developed	100%	48%	100%	50%[23]
Value of Flexibility				
Increase in ENPV		7,518		3,511
% Increase in ENPV		-79%		-18%
Decrease in St Dev		-11,105		-7,281

Table 4: Comparison of Static versus Flexible Cases ($ Thousands)

The first observation is that the options add significant value to both programs' ENPV – approximately $7.5 million for the single building program and $3.5 million for the phased building program. One explanation for the difference in added value between the two programs is that the single building is poised to better take advantage of scenarios in which the value of built property is exceptionally high. It represents a larger exposure to market conditions at any given time, and thus the ability to wait until conditions are right enables it to better take advantage of high value and growth.

The opportunity to delay the projects also adds significant downside mitigation to both programs, as evidenced by the percentage of cases in which the project is never developed. Also noteworthy is the dramatic decline in standard deviation for both phases.[24] Both programs are significantly less risky. However, the decline in risk comes at a cost. In exchange for a significantly reduced downside, the median NPV is lower for the flexible case than for the static case: $4.4 million versus $3.1 million for the single building program, and $14.6 million versus $6.8 million for the phased building program. This is because, as we introduce the option to defer, there are a number of scenarios in which the project is never developed, but for which there would have been a positive NPV. This number of

---

[23] The first phase is developed 50% of the time, whereas the second phase is developed 46% of the time.
[24] Note that the single program now has a lower standard deviation than the phased program, although the standard deviation as a percentage of ENPV is still higher.

abandoned projects is higher for the phased building program due its higher average per square foot construction cost.

Again, we compare the static and flexible cases by looking at VARG curves for each program. The circles on each graph represent where the options to defer or abandon development add value or lead to a loss[25]. **Figure 10** and **Figure 11** both show a substantial rightward shift of the VARG curve in the lower range of returns, reflecting significantly reduced downside risks for the flexible case. For instance, while there is a 10% chance that the project NPV will be between negative $36.5 million and negative $89.6 million in the static case, the project NPV will be between negative $56.7 million and *positive* $30,000 in the flexible case with the same probability. For the phased building program, there is a 10% chance that the project NPV will be between negative $28.0 million and negative $82.3 million in the static case. However, the project NPV will be between negative $38.5 million and *positive* $147,000 in the flexible case with the same probability.

The area between the two curves in the lower left corners of the same figures represents the value of the option. The VARG for the flexible case spikes on the right edge of this area. This represents the large number of cases in which development never occurs because the built asset value never exceeds its hurdle value (V^*). Due to this phenomenon, there are a number of cases where the static ENPV is greater than the flexible ENPV for both programs. These cases are represented by the area where the flexible VARG is greater than the static VARG. The area between these two curves represents the cost of the option. As the area labeled *Option Value* is greater than the area labeled *Option Cost*, the net impact of the option is positive. Thus, overall, the flexible case yields a higher ENPV than the static case for both programs.

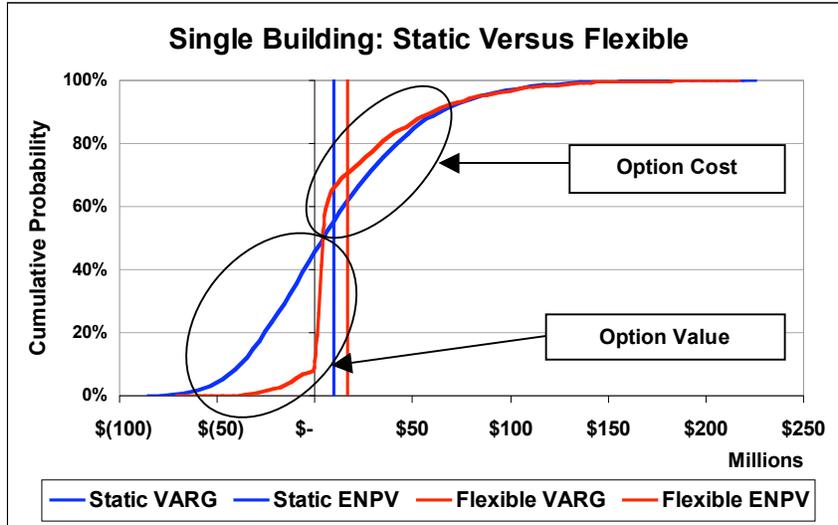[25] This is interpreted as the "cost of the option".

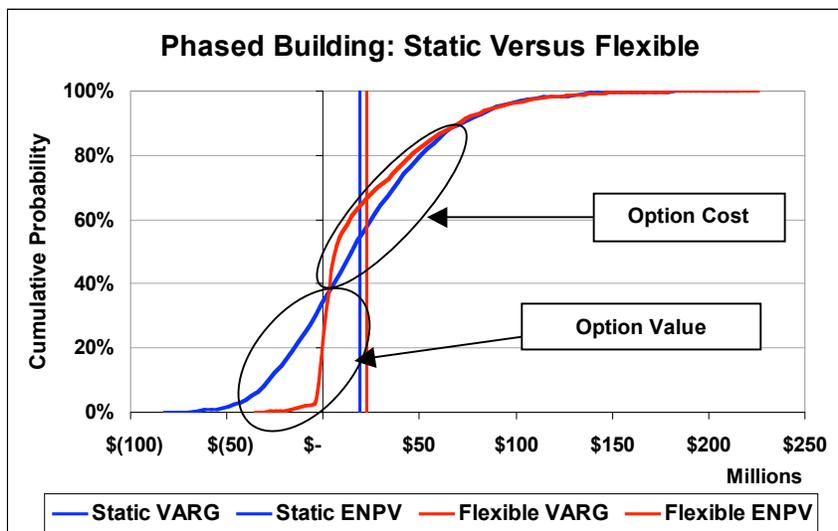Figure 10: VARG Curves – Static versus flexible single building program

Figure 11: VARG Curves – Static versus flexible phased building program

6.2.4. Determining the Highest and Best Use of the Site

The highest and best use of the site is the phased development program. The VARG curve for the flexible single and phased building programs, both with flexibility, are shown in **Figure 12**. The figure shows that the ENPV for the phased building program is higher than that of the single building program. The circled area represents the cumulative probability

for which the single building program is expected to have a higher NPV than the phased building program (close to where the ENPV is zero). At this point, both VARG curves spike due to the instances when development never occurs. As mentioned, abandonment occurs more frequently for the phased building program than the single building program.
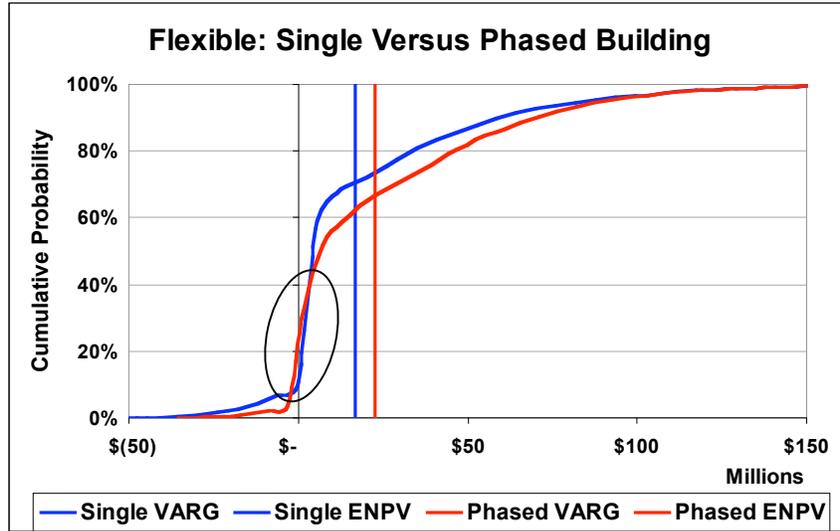

Figure 12: VARG Curves – Single versus phased building programs with flexibility

The histogram for the flexible phased program is shown in **Figure 13**. The spike in the distribution reflects the high percentage of cases in which development never occurs. The distribution also reflects the low number of possible negative outcomes for this scenario.
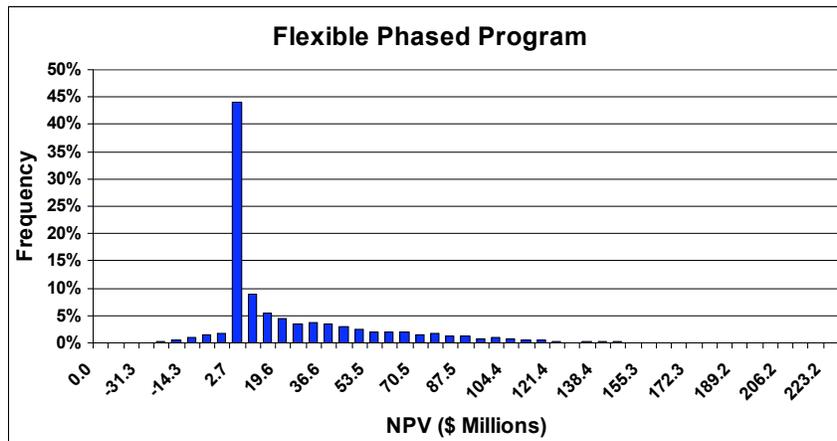

Figure 13: Histogram – Flexible phased building program

6.3. Case Conclusion

Based on the results of step three, we conclude that phasing the project still presents the highest and best use of the site. However, we are able to get a much better sense of the risk exposure of each program using the NPV statistics, the histograms, and the VARG curves. Furthermore, we are able to assess the value of the option to defer development for up to nine years, as well as the value of the option to phase the project as opposed to constructing the project all at once in an untested market.

7. Conclusion

The aim of this thesis is to facilitate the real estate industry's adoption of real options analysis by providing a simple and transparent model that not only serves as a proof of concept for real options analysis, but values the relative risks and returns associated with the real options that developers commonly face in practice. By providing an easy-to-use model that rigorously quantifies the value of options in a project, developers will be better equipped to assess the risks and returns associated with that project and use that assessment as a basis for decision-making. This is especially true in the current environment in which new financial instruments such as real estate derivatives allow for a more efficient and transparent real estate market.

The interviews that we conducted with industry professionals helped us understand the decision making process and shape our model to dovetail with current valuation techniques. We learned that a developer's job is to assess and mitigate risk. We also learned that developers implicitly value real options via intuition and judgment by adjusting the hurdle rates or return requirements for any given project depending on the amount of options available - more options serve to reduce risk and therefore lower return requirements. Further, the decision rules that developers employ when deciding whether or not to start a project are fairly standard and entrenched.

The real options model presented in this thesis is hybrid of the economic and engineering real options methodologies. We base our "go / no go" decision rules on the traditional NPV metric and the Samuelson McKean (S-M) hurdle value. The S-M formula, which is part of the economic methodology, is also used to calculate the abandonment value of land. A key element of the economic approach that we leave out is the binomial options pricing model. Though this model has been applied to development projects in the past, the real estate community has not adopted it because it is unfamiliar and does not conform to current valuation techniques.

The broad framework of our model is based on the engineering approach. Specifically, it uses Monte Carlo simulations to incorporate uncertainty and employs performance metrics such as Expected Net Present Value (ENPV), Histograms, and Value at Risk and Gain (VARG) curves. This approach allows us to integrate the real options model with the traditional discounted cash flow calculations that most developers are familiar with, thus allowing for easy adoption of the model. The main component of the engineering methodology that we leave out is the adaptive OFAT method. This procedure can be time consuming and must be customized for each project. Further, because the decision rules used in development tend to be standardized, this procedure is not necessary and may act as a barrier to adoption of the model.

The resulting model is targeted to the following real options scenarios, both of which are common in real estate development: the option to phase a project, and the option to choose among multiple uses. Both modes of operation also include the option to defer development, and thus implicitly calculate a compound option. The phasing case study shows that the project with options is both less risky and yields a higher expected net present value than the project without options.

The model should not be used as a complete land residual model due to the simplifications we made. However, it does serve as a proof of concept for the use of real options analysis for real estate development projects. Further, it is useful in measuring a project's relative value and risk with and without options.

Bibliography

Black, F. and M. Scholes (1973). "The Pricing of Options and Corporate Liabilities", *Journal of Political Economy* 81: 637-659.

Bodie, Kane, and Marcus (2002). *Investments*, 5th ed., McGraw-Hill / Irwin; New York, NY.

Brealey, R., S. Myers, and F. Allen (2006). *Principles of Corporate Finance,* 8th ed., McGraw-Hill / Irwin; New York, NY.

Cardin, Michel-Alexandre (2007). "Facing Reality: Design and Management of Flexible Engineering Systems", MS Thesis, MIT Engineering Systems Division, Cambridge, MA.

Childs, P., S. Ott and T. Riddiough (2002). "Optimal Valuation of Claims on Noisy Real Assets: Theory and an Application," with Paul Childs and Steve Ott, *Real Estate Economics,* Fall 2002, Vol. 30 (3), pp. 415-444.

Copeland, T. E. and V. Antikarov (2003). *Real Options: A Practitioner's Guide*, Thomson Texere: New York, NY.

Copeland, T. E., Weston, J. F., and Shastri, K. (2003). *Financial Theory and Corporate Policy*, 4th ed., Addison Wesley: White Plains, NY.

Cox, J. C., Ross, S. A., and Rubenstein, M. (1979). "Options Pricing: A Simplified Approach", *Journal of Financial Economics*, 7(3): 229-263.

de Neufville, R., Scholtes, S. and Wang, T. (2006). "Valuing Options by Spreadsheet: Parking Garage Case Example", *ASCE Journal of Infrastructure Systems*, 12, 2, pp. 107-111. http://ardent.mit.edu/real_options/Real_opts_papers/Garage Case_Tech_Note Draft Final January.pdf

de Neufville, R. (2006). "Lecture Notes", ESD.70J, Engineering Systems Division, Massachusetts Institute of Technology, Cambridge, MA.

Geltner, D. and N.G. Miller (2007). *Commercial Real Estate Analysis and Investments*, 2nd ed., Thomson South-Western; Mason, OH.

Geltner, D., T. Riddiough, and S. Stojanovic (1996). "Insights on the Effect of Land Use Choice: The Perpetual Option on the Best of Two Underlying Assets", *Journal of Urban Economics* 39(1): 20-50.

Hengels, A. (2005). "Creating a Large Practical Model Using Real Options to Evaluate Large-Scale Real Estate Development Projects", MSRED Thesis, MIT Center for Real Estate, Cambridge, MA.

Higgins, Robert C. (2007). *Analysis for Financial Management*, 8th ed., McGraw-Hill / Irwin; New York, NY.

Jenter, D. (2007). "Lecture Notes", Arundel Partners and Real Options, MIT Sloan Finance Theory II 15.402: Spring 2007.

Kang, J. (2004). "Valuing Flexibility in Large-Scale Real Estate Development Projects", MSRED Thesis, MIT Center for Real Estate, Cambridge, MA.

Luccioni, L. X., and R. D. Espinoza (2005). "An Approximate Solution for Perpetual American Option with Time to Build: the Value of Environmental Remediation Investment Projects" http://realoptions.org/papers2005/Espinoza_An%20Approximate%20Solution%20for%20Perpetual%20American%20Option.pdf

McDonald, R. L. (1998). "Real Options and Rules of Thumb in Capital Budgeting", Finance Department, Kellogg School, Northwestern University, March.

Quigg, L. (1993). "Empirical Testing of Real Option-Pricing Models", *Journal of Finance* 48(2): 621-40, June.

Sing, T.F. and K. Patel (2001). "Evidence of irreversibility in the UK property market", *Quarterly Review of Economics and Finance*, 41(3): 313-334.

Teague, Neel (2007). Personal Interview, June 26.